1 Charging a mobile phone with small PV panel

1.1

The example here uses 1800 mAh capacity battery. This *capacity* rating means that the battery can deliver

1800mA for 1h, or
900 mA for 2h, or
600mA for 3h, etc…

However, this rating only tells us how much CHARGE the battery can deliver. It does not tell us the energy. How do we go about solving the problem?

We know that:

$$P = Vi$$

P is power, *i* is current and *V* is voltage.

We also know that:

$$E = Pt$$

Where E is energy and t is time. Hopefully this equation is well known by you.

With these two equations we can write:

$$E = Vit$$

Notice that we have ***current x time*** (it) in the equation above, which we can be defined as the *capacity* of the battery, i.e. the total charge it can deliver. If you look at the units used for current (Coulomb per second) and time (in seconds), this is clearly evident.

$$[i] = C/s$$

$$\therefore [it] = \frac{C}{s}s = C$$

To know the energy E which the battery can deliver, we are missing the voltage V. A typical lithium ion battery for mobile devices works at *3.7 V.*

Knowing the capacity=1800mAh and the Voltage=3.7V we can work out the energy contained in the battery.

$$E_{bat} = Vit$$

$$V = 3.7V$$
$$it = 1800mAh$$
$$\therefore$$
$$E_{bat} = Vit = 3.7V \times 1800mAh$$
$$= 3.7 \times 1800mVAh$$
$$= 6660mVAh$$

$$E_{bat} = 6660mVAh$$

Are you confused by the units?

You are well aware that a good measure of electrical energy for practical purposes is not Joules but something like kWh or Wh. Do you remember what these values actually mean? Remember that the units of power are W or J/s.

So why is $P = Vi$?

We have looked at the units of i, so let us look at the units of V and the definition of voltage.

"The voltage between two points is equal to the work done per unit of charge against a static electric field to move the charge between two points and is measured in units of volts (a joule per coulomb)."

i.e.

$$[V] = J/C$$
$$\therefore [P] = [V][i]$$
$$[P] = \frac{J}{C}\frac{C}{s} = \frac{J}{s} = W$$

If we go back to our answer of the energy stored in the battery of E=6660mVAh we now know we can write:

$$E = 6660 mVAh$$
$$= 6.66 VAh$$
$$[VAh] = \frac{J}{C}\frac{C}{s}h$$
$$= \frac{J}{C}h$$
$$= Wh$$

That is:

$$E = 6.66 Wh$$

1.2

Knowing the energy that can be stored in the battery we now want to work out how long our solar panel will take to charge the battery.

Our module has an area of 50cm^2 and an efficiency of 15%, that is, it converts 15% of the incident power onto in into electricity.

Typically, a module's characteristics are defined under standard conditions. Normally the standard incident power (or peak power) is P_{sta}=1000W/m^2.

To know the actual incident power on our module (P_{inc}), we need to take into account its area A:

$$P_{sta} = 1000W / cm^2 = 100mW / cm^2$$

$$P_{inc} = P_{sta} \times A$$
$$A = 50cm^2$$
$$\therefore P_{inc} = 100\frac{mW}{cm^2} \times 50cm^2$$
$$= 5000mW$$
$$P_{inc} = 5W$$

Knowing the incident power on the module, and the module's efficiency, we can now calculate how much power the module produces (P_{out}).

$$\eta = \frac{P_{out}}{P_{inc}}$$
$$\therefore P_{out} = P_{inc}\eta$$
$$= 5W \times 15\%$$

$$P_{out} = 0.75W$$

To charge the battery, the module has to deliver 6.66Wh. Thus the piece of information missing to complete our task is; "that is how long it takes to charge the battery?" can be found by rearranging the equation below and calculating:

$$E = Pt$$
$$\therefore$$
$$t = \frac{E}{P}$$

$$E_{bat} = 6.66 Wh/cycle$$
$$P_{out} = 0.075W$$

$$t = \frac{6.66Wh}{0.75W} = 8.88\, h/cycle$$

$$t = 8.88\, h/cycle$$

Always take notice of how the units play out. In this case, the W cancel to leave time in h.

1.3

The question is best worded in terms of how often could you could charge your phone.

We have an annual insolation of:

$$E_{insolation} = 1900kWh / m^2 / year$$

That is, over the course of one year, 1900kWh of energy is incident on 1m^2 area.

We must now understand how much of the solar insulation we can convert into electrical energy, and for that we require the module efficiency η=15% and module area A=50cm^2.

Firstly the insolation on an area of 50cm^2 (the module area) is:

$$E_{insolation,\mathrm{mod}} = 1900\frac{kWh}{m^2.year}\times 50cm^2$$
$$= 1900\frac{kWh}{m^2.year}\times 0.005m^2$$
$$E_{insolation,\mathrm{mod}} = 9.5kWh/year$$

Next, the module can convert 15% of this incident insolation to useful electrical energy:

$$E_{prod,\mathrm{mod}} = \eta E_{insolation,\mathrm{mod}}$$
$$= 15\% \times 9.5 kWh/year$$
$$E_{prod,\mathrm{mod}} = 1.4 kWh/year$$

We now know how much energy the module can deliver over one year, and we know the energy we require to charge the battery. As such, we can now work out how many n times we could charge the battery with our PV charger over the course of one year.

$$E_{prod,\mathrm{mod}} = 1.4 kWh/year$$
$$E_{bat} = 6.66Wh/cycle$$

$$\therefore n = \frac{E_{prod,\mathrm{mod}}}{E_{bat}}$$
$$= \frac{1.4kWh/year}{6.66Wh/cycle} = \frac{1400Wh}{6.66Wh}$$
$$= 210 cycle/year$$

$$n = 210 cycle/year$$

2

2.1

In this question we have insolation defined using other units: 1.5kWh/Wp/year, that is the amount of energy a system will produce over the course of the year for every W of power installed.

Thus a P_{Wp}=200Wp system will over the course of a year produce:

$$E_{insolation} = 1.5kWh/W_p/year$$
$$P_{Wp} = 200W_P$$

$$E_{prod} = E_{insolation}\times P_{Wp}$$
$$= 1.5kWh/W_p/year \times 200W_P$$
$$= 1.5\times 200kWh/year$$

$$E_{prod} = 300kWh/year$$

2.2

To discuss the savings we have to think carefully about what how "self-consumption" can occur. You will notice that here we have deliberately chosen a small size system of 200W$_p$.

Let us assume that our baseload is over 200W whenever the PV system is active (i.e. during daylight hours). As such we can then assume that all the electricity produced by the modules (E_{prod}) is consumed and thus is a saving from the grid (E_{saved}), i.e. we are not consuming electricity costing for each unit of energy 0.15€/kWh.

$$E_{prod} = E_{save}$$
$$\because E_{prod} = 300kWh/year$$
$$\therefore E_{save} = 300kWh/year$$

As a PV system is typically guaranteed now for 30 years;

$$E_{save} = 300\ kWh/year\ \times 30year$$
$$E_{save} = 9000\ kWh$$

Now the question states that the cost of electricity goes up exponentially with an increment of 2% annually.

A system with an exponential growth described by a % annual increase x, the value for the year n can be found using the expression:

$$y(n) = y(n=1) \times \left(1 + x^{n-1}\right)$$

Or in our case:

$$T_{c€/kWh}(n) = 15c€/kWh \times (1+2\%)^{n-1}$$
$$T_{c€/kWh}(n) = 15c€/kWh \times (1.02)^{n-1}$$

For your calculations, you will need to build a table showing the cost of electricity for each year.

Year n	Tariff c€/kWh	Year n	Tariff c€/kWh
1	15	16	
2	=15c€/kWh*1.02^n=15.3	17	
3	15.61	18	
4		19	
5		20	
6		21	
7		22	
8		23	
9		24	
10		25	
11		26	
12		27	
13		28	
14		29	
15		30	26.64

Considering that the self-consumption for each year remains constant, but the cost of electricity increases, to find the total saving we must sum the savings S for each individual year.

$$S(n) = T(n)E_{save}$$
$$= 15c€/kWh \times (1.02)^{n-1} \times 300kWh/year$$

And so the total savings for the 30 years is:

$$S(n) = T(n)E_{save}$$
$$= 15c€/kWh \times (1.02)^{n-1} \times 300kWh/year$$
$$S_{total} = \sum_{n=1}^{n=30} S(n) = \sum_{n=1}^{n=30} 15c€/kWh \times (1.02)^{n-1} \times 300kWh/year$$

Because the value of the quantity of electricity self-consumed each year does not vary, we can write:

$$S_{total} = 300kWh/year \sum_{n=1}^{n=30} 15c€/kWh \times (1.02)^{n-1}$$

And then performing the calculation:

$$\sum_{n=1}^{n=30} 15c€/kWh \times (1.02)^{n-1} = 6.08€$$
$$\therefore S_{total} = 300kWh/year \times 6.08€/kWh$$
$$= 1824€$$

2.3

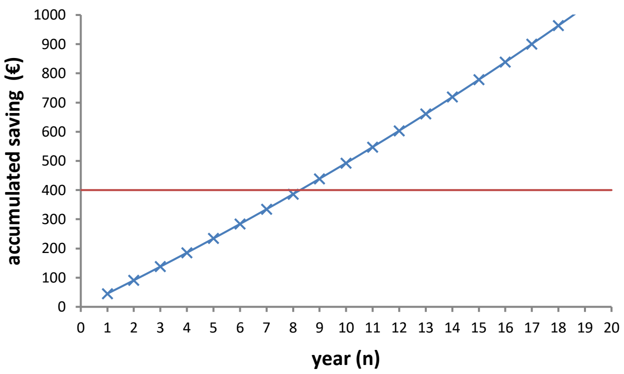
One way of looking at this problem is to visualise how long it takes for the savings to be savings are equal to the total cost of installation C_{inst}.

$$\sum_{n=1} S(n) = C_{inst}$$

The total investment for a 200W_p system, is if the installation cost is 2€/W_p;

$$C_{inst} = P_{Wp}C_{Wp}$$
$$= 200W_p \times 2€/W_p$$
$$= 400€$$

The accumulated saving year on year can be calculated and the easiest way to visualise it is in a graph.



…where the blue line is the year-on-year accumulated saving, and the red line the cost of installation.

From the graph we can see that the payback time is just over 8 years.

2.4

The points to think about are:

- Power installed, and
- What is the expected temporal profile for the power consumption?
  - Is it continuous?
  - Can we consume everything we produce all the time?
- Under what circumstances is the "self-consumption" law a viable business option?

3.

3.1

What we know:

$$\eta_{\text{mod}} = 15\%$$
$$C_{BOS,area} = 200€/m^2$$
$$C_{Wp} = 1€/W_p$$

That is, the module efficiency η_{mod}, the cost of installation per unit area C_{area} and the cost of installation per unit of power installed C_{Wp}.

To a first order approximation, the BOS scales with the area of installation, and so to decrease this associated cost, it is advantageous to minimise the area of installation. This is only possible (if we maintain the same power) by increasing the efficiency of the modules so that per unit area they produce more.

As such we need to know how much power P_{area} we are installing per unit area and its associated cost $C_{PV,area}$.

The power per unit area $W_{P,\ area}$ can be worked out by taking into consideration the module efficiency η_{mod} and understanding that the W_P rating is the power a module will produce under standard conditions, i.e. when $P_{inc.,sta} = 1000W_P/m^2$:

$$W_{P,area} = \eta_{\text{mod}} \times P_{inc.,sta}$$
$$= 15\% \times 1000W_P/m^2$$

$$W_{P,area} = 150W_P/m^2$$

We now know how much power is installed per unit area $W_{P,area}$, and we know the cost of each unit of installed power $C_{Wp} = 1€/W_p$, and thus can work out the cost of the modules per unit area:

$$C_{\text{mod},area} = W_{P,area} \times C_{Wp}$$
$$= 150 W_P / m^2 \times 1€ / W_p$$

$$C_{\text{mod},area} = 150€ / m^2$$

And so the total installation cost per unit area is the sum of the BOS and the module installation:

$$C_{area} = C_{\text{mod},area} + C_{BOS,area}$$
$$= 150€ / m^2 + 200€ / m^2$$

$$C_{area} = 350€ / m^2$$

3.2

In this case we have a technology who's module cost is significantly lower, at $C_{\text{mod},area} = 20€ / m^2$ rather than $150€ / m^2$. For this new module technology to be competitive, its cost has to be less or equal than that we calculated beforehand. However, because the module cost is lower per unit area, we can get away with installing a greater area of modules. As such, we have to revert to calculating on a power basis C_{Wp} and not area basis.

Knowing the power installed per unit area $W_{P,area}$ and the total cost of per unit area C_{area} we can work out the cost per unit of power:

$$C_{Wp} = \frac{C_{area}}{W_{P,area}}$$
$$= \frac{350€ / m^2}{150 W_P}$$
$$C_{Wp} = 2.3€ / W_P$$

That is, our new PV module technology cost per unit of power has to be less than:

$$2.3€ / W_P \geq C_{Wp,new}$$

Now we can work back on the basis of how the parameters are defined:

$$C_{Wp,new} = \frac{C_{area,new}}{W_{P,area,new}}$$
$$\because C_{area} = C_{\text{mod},area} + C_{BOS,area}$$
$$\therefore C_{Wp,new} = \frac{C_{\text{mod},area,new} + C_{BOS,area}}{W_{P,area,new}}$$

And remembering that we know:

$$C_{BOS,area} = 200€ / m^2$$
$$C_{\text{mod},area,new} = 20€ / m^2$$

Next we can substitute in for $W_{P,area,new}$:

$$\because W_{P,area,new} = \eta_{\text{mod}} \times P_{inc.,sta}$$
$$\therefore C_{Wp,new} = \frac{C_{\text{mod},area} + C_{BOS,area}}{\eta_{\text{mod}} \times P_{inc.,sta}}$$

We now have an expression with only one unknown (by definition $P_{inc.,sta} = 1000 W / m^2$), that is efficiency. We can therefore now use the condition for minimum cost to define the minimum efficiency requirement:

$$\because C_{Wp,new} \leq 2.3€ / W_P$$
$$\therefore \frac{C_{\text{mod},area} + C_{BOS,area}}{\eta_{\text{mod}} \times P_{inc.,sta}} \leq 2.3€ / W_P$$

$$\frac{C_{\text{mod},area} + C_{BOS,area}}{2.3€ / W_P \times P_{inc.,sta}} \leq \eta_{\text{mod}}$$

$$\eta_{\text{mod}} \geq \frac{20€ / m^2 + 200€ / m^2}{2.3€ / W_P \times 1000 W_P / m^2}$$
$$= \frac{20 + 200}{2.3 \times 1000}$$
$$\eta_{\text{mod}} \geq 0.096$$

Moral of the story is: even if we someday invent a very cheap way to manufacture modules, we still have to find ways of decreasing the cost of installation of the supporting systems.

4.

$$E_{cons} = 50\times10^{12} Wh/year$$
$$= 5\times10^{10} kWh/year$$

$$P_{inst} = \frac{E_{cons}}{E_{prod,Wp}}$$
$$= \frac{5\times10^{10} kWh/year}{1.5 kWh/W_P/year}$$
$$= 3.3\times10^{10} W_P$$

$$W_{P,area} = \eta_{\text{mod}} \times P_{inc.,sta}$$
$$= 15\% \times 1000 W_P/m^2$$
$$= 150 W_P/m^2$$

$$A = \frac{P_{inst}}{W_{P,area}}$$
$$= \frac{3.3\times10^{10} W_P}{150 W_P/m^2}$$
$$= 2.2\times10^{8} m^2$$
$$= 220 km^2$$

Think about how this area compares to the area of Portugal? It is a small fraction.

However, what would the investment cost be? How does this compare to GDP?